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CUBIC SPLINES IN CLUTTER MODELING.(U)

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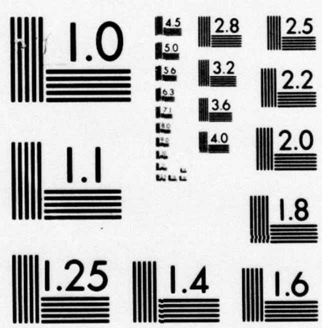

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DATE: 11 18 November 1976

TO: R. B. Thompson 12 5p.

FROM: W. G. Peoples

SUBJECT: 6 Cubic Splines in Clutter Modeling,

REFERENCES: 1) Raytheon Company Missile System Division BR-9254 "TAGSEA Program Final Report (27 August 1967).

2) R. C. Davis, P0008716, "Critical Review of TAGSEA Clutter Models, Final Report".

Naval Sea Sysyems Command.
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INTRODUCTION

In developing clutter models, Raytheon used smoothing cubic splines to obtain analytical expression for the clutter amplitude distribution functions. Volume III of Reference 1 discusses Raytheon's application of cubic splines. This application is believed to be unique. This report has been prepared to describe in detail the development of the smoothing cubic spline. An explanation of the large oscillations in the error obtained in the fit or data match for large clutter values is given, and a method that would avoid the oscillations is discussed.

DISCUSSION

Smoothing cubic splines are useful tools to reconstruct a continuous curve from discrete noisy data obtained from observations of a physical process in which one is interested only in a curve fit whose ordinate, slope, and curvature are continuous over the entire range of abscissa values. A detailed exposition of the smoothing cubic spline developed in Section 1.1.2 of Appendix B in Volume III is given below to clarify the procedures used by Raytheon. First define $H(x) = \mathrm{Log}_{10} \left|1-F(x)\right|$, where $F(x)$ is the cumulative distribution of the observed random variable x. The sequence $\left|x_i\right|$, $i=1,2,\ldots, N$ of observations is given, and the value of the histogram at the observed $x = x_i$ is denoted by $H(x_i)$. If $t = \log_{10} (x/10)$, a cubic spline $S(t)$ can be determined in the following manner as a continuous smoothed fit to $H(x)$: Denoting by t_{k_1}, t_{k_2}, t_{k_3}, and t_{k_4} four as yet undetermined t values out of the sequence $\left|t_i\right|$, $i = 1,2,\ldots,N$ an $S(t)$ defined for $t_{k_1} \le t \le t_{k_4}$ is to be found that has the following properties: In each of the three t intervals $S(t)$ is a cubic polynomial in t, so that in general three separate cubics can be constructed. These cubics are constrained so that $S(t_{k_j}^-) = S(t_{k_j}^+)$; $S'(t_{k_j}^-) = S'(t_{k_j}^+)$, and $S''(t_{k_j}^-) = S''(t_{k_j}+)$ for $j \equiv 1$ and 2. In effect then the desired function $S(t)$ will be continuous everywhere and possess continuous first and second derivatives in the interval $t_{k_1} \le t \le t_{k_4}$.

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Moreover, subject to these constraints, S(t) is to be chosen to minimize the quantity E, where

$$E = \sum_{i=1}^{N} W_i \left[H(x_i) - S(t_i)\right]^2$$

and in which

$$t_i = 10 \log_{10} (x_i/10) \quad , \quad i = 1,2,\ldots,N$$

$$w_1 = \left(t_2 - t_1\right)/\left(t_N - t_1\right)$$

$$w_i = \left(t_{i+1} - t_i\right)/\left(t_N - t_1\right) \quad , \quad i = 2,3,\ldots,N$$

$$w_N = \left(t_N - t_{N-1}\right)/\left(t_N - t_1\right)$$

The four members of the sequence $\left\{t_{k_j}\right\}$ are termed the knots of the spline, i.e., the points at which the various continuity properties must hold. If S(t) is to encompass the entire range of values of t, $t_{k_1} \equiv t_1$, and $t_{k_4} \equiv t_N$. Before discussing the manner in which t_{k_2} and t_{k_3} are chosen so as to minimize E, the way the three cubics are obtained for fixed knots so as to minimize E will be shown. There are several ways to carry out the analysis; the method which leads to the simplest calculations will be shown.

First the truncated power function t_+^3 is defined as

$$t_+^3 = \begin{cases} t^3 & , \quad t>0 \\ 0 & \quad\;\; t\le 0 \end{cases}$$

Every cubic spline S(t) defined over $t_{k_1} \le t \le t_{k_4}$ with interior knots t_{k_2} and t_{k_3} has a unique representation

$$S(t) = p_3(t) + c_1(t-t_{k_2})_+^3 + c_2(t-t_{k_3})_+^3 \; , \; t_{k_1} \le t \le t_{k_4}$$

where $p_3(t)$ is a cubic polynomial.

Hence,

$$S(t) = S_1(t) = a_1(t-t_{k_1})^3 + a_2(t-t_{k_1})^2 + a_3(t-t_{k_1}) + a_4,$$

$$t_{k_1} \le t \le t_{k_2}.$$

$$S(t) = S_1(t) + c_1(t-t_{k_2})^3 \quad , \quad t_{k_2} \le t \le t_{k_3}.$$

$$= S_1(t) + c_1(t-t_{k_2})^3 + c_2(t-t_{k_3})^3 \quad , \quad t_{k_3} \le t \le t_{k_4}.$$

Hence the criterion function that is to be minimized has the form

$$E = \sum_{j=1}^{k_2} W_j \left[H(x_j) - a_1(t_j-t_{k_1})^3 - a_2(t_j-t_{k_1})^2 - a_3(t_j-t_{k_1}) - a_4\right]^2$$

$$+ \sum_{j-_2}^{k_3} W_j \left[H(x_j) - c_1(t_j-t_{k_2}) - a_1(t_j-t_{k_1})^3 - a_2(t_j-t_{k_1})^2\right.$$

$$\left. - a_3(t_j-t_{k_1}) - a_4\right]^2$$

$$+ \sum_{j-k_3}^{N} W_j \left[H(x_j) - c_2(t_j-t_{k_3})^3 - c_1(t_j-t_{k_2})^3 - a_1(t_j-t_{k_1})^3\right.$$

$$\left. -\ldots -a_4\right]^2 .$$

The determination of c_1, c_2, a_1, a_2, a_3, a_4 is made easiest by solving for c_1 as a linear combination of a_1, a_2, a_3, a_4 in the equation $\frac{\partial E}{\partial c_1} = 0$ as one of the six minimizing equations. Specifically

$$c_1 = \frac{\sum_{j=k_2}^{k_3} W_j(t_j-t_{R_2})^3 \left[H(x_j) - a_1(t_j-t_{k_1})^3 - a_2(t_j-t_{k_1}) - a_3(t_j-t_{k_1}) - a_4\right]}{\sum_{j-k_2}^{k_3} W_j(t_j-t_{k_2})^6}$$

The equation $\frac{\partial E}{\partial c_2} = 0$ can be used to express c_2 as another linear combination of a_1, a_2, a_3, a_4. Then the equations $\frac{\partial E}{\partial a_1} = \frac{\partial E}{\partial a_2} = \frac{\partial E}{\partial a_3} = \frac{\partial E}{\partial a_4} = 0$

yield a linear set of equations requiring only the inversion of a 4x4 matrix, or the equations can be solved directly by inversion of a 6x6 matrix. Finally, the location of the variable knots t_{k_2} and t_{k_3} is determined by those values which minimize E. A variable knot computer program was used by Raytheon to determine the placement of t_{k_2} and t_{k_3} which minimize E. The computer program is a standard routine in the International Mathematical and Statistical Libraries (IMSL) called ICS VKU.

An improved fit would have been obtained by fitting a smoothing cubic spline directly to the histogram F(x), i = 1,2,..., N and imposing the boundary conditions on the spline fit S(x) that $S(x_1) = S'(x_1) = 0$ and $S(x_N) = 1$, $S'(x_N) = 0$.

With the relation $t = 10 \log_{10} (\frac{x}{10})$ large errors in the fits obtained to the function $G(t) = \log_{10} \left|1-F(x)\right|$ are to be expected for large values of t. The discussion below gives the reason for the large oscillations in the error obtained in every fit for large values of t. From the function $H(x) = \log_{10} \left|1-F(x)\right|$, the relation

$$H'(x) = \frac{-\left[\log_{10} e\right] f(x)}{1 - F(x)}$$

can be obtained where f(x) denotes the probability density function of x. By a double application of L'Hospital's rule, the conclusion that $\lim_{x\to\infty} H'(x) = -\infty$ can be reached. Since $G'(t) = \frac{x}{10 \log_{10} e} H'(x)$,

$\lim_{x\to\infty} G'(t) = -\infty$. Moreover the $\lim_{x\to\infty} H(x) = \lim_{x\to\infty} G(t) = -\infty$.

Since a fit G(t) is to be obtained by a smoothing cubic spline which possesses finite ordinates and slopes everywhere, it is to be expected that large oscillations would occur in the fit for large values of t. This is shown in the error graphs on pages B-27, B-31, B-35, etc. of Volume III.